

# **Is the optimum XY spacing of the Generalized Reciprocal Method (GRM) constant or variable? \***

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## **Abstract**

The Generalized Reciprocal Method (GRM) has been proposed for mapping of subsurface structures with lateral variations. This method depends mainly on a single parameter called the optimum XY distance. At the optimum XY separation, forward and reverse rays to each geophone emerge from near the same point on the refractor. This value is based on heuristic determination and it is always a doubtful matter. The uncertainties of the optimum XY spacing will be discussed on several synthetic models.

## **Key words**

Optimum XY, dipping refractor, faulted layer, irregular refractor, undulating topography.

## **Introduction**

The Generalized Reciprocal Method (GRM); introduced by Palmer (1980), uses separate imaginary points on the travel time curves to estimate the optimum XY spacing. The optimum XY spacing is defined by Palmer (1980) as "at the optimum XY separation, the forward and reverse rays to each geophone emerge from near the same point on the refractor". This method has acquired a considerable support of some geophysicists who work in this field. On the other hand, the optimum XY spacing is criticized by many authors, e.g., Leung (2003 & 1995), Sjorgen (2000) and Whiteley (2006).

In this study, some simple models will be reviewed and discussed in order to show the uncertainties of the optimum XY spacing. The rays recorded from refracting interfaces with variable depth and velocity will show that there is no a single value of "optimum XY".

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## Main parts of the GRM

The GRM has two functions, the velocity analysis and the time-depth functions. The velocity analysis function is given by:

$$t_v = (T_{AY} - T_{BX} + T_{AB}) / 2$$

The optimum XY showing a maximum smoothness (or a less structure) will confirm to the geophysical assumptions that valid for most geology

While, the time-depth function is:

$$T_G = [T_{AY} + T_{BX} - (T_{AB} + XY / V'_n)] / 2$$

The term  $V'_n$  is the apparent refractor velocity determined from the velocity analysis function. It is clear that these two functions depend mainly on optimum XY spacing.

In the following, four cases will be discussed to show the absence of the optimum XY values and consequently the limitations of the GRM technique.

### First case

#### Plane dipping refracting interface:

This model represents a two layers case with a dipping refracting interface (Fig.1). The dip angle is about 5 degrees. Applying the definition of the optimum XY spacing as given by Palmer (1980) on this simple model, the following will be found:

1. There is no single value from which the rays emerge from the same point (*or from near the same point*) on the refractor, instead we have variable XY spacing (Fig 1a). This indicates that there is no single value from which the rays emerge from the same point on the refractor (no optimum XY spacing).
2. At three different points on the refractor G', G and G'', it will be found that there are three different corresponding XY spacing (three optimum XY); in the middle part XY value equals to 10.81 m, while this value is equal to 6.1 m at X'Y' and 14.89 m at X''Y'' at the outer parts of the model (Fig.1b). This means that if the rays emerge from the same point on the refractor, then they will have different XY values varying from 6.1 m to 14.89 m. Which one of these value can be used as optimum spacing?
3. If the XY spacing of the middle part (equals to 10.81 m) is selected as optimum spacing, then the X'Y' and X''Y'' rays will emerge from two different points (not from near the same point) on the refractor (Fig. 1c).

4. In case of the XY spacing of 10.81 m, the depth from which the rays emerge to X'Y' will be below the refractor, while the rays emerging to X''Y'' will be above this refractor (Fig. 1d).

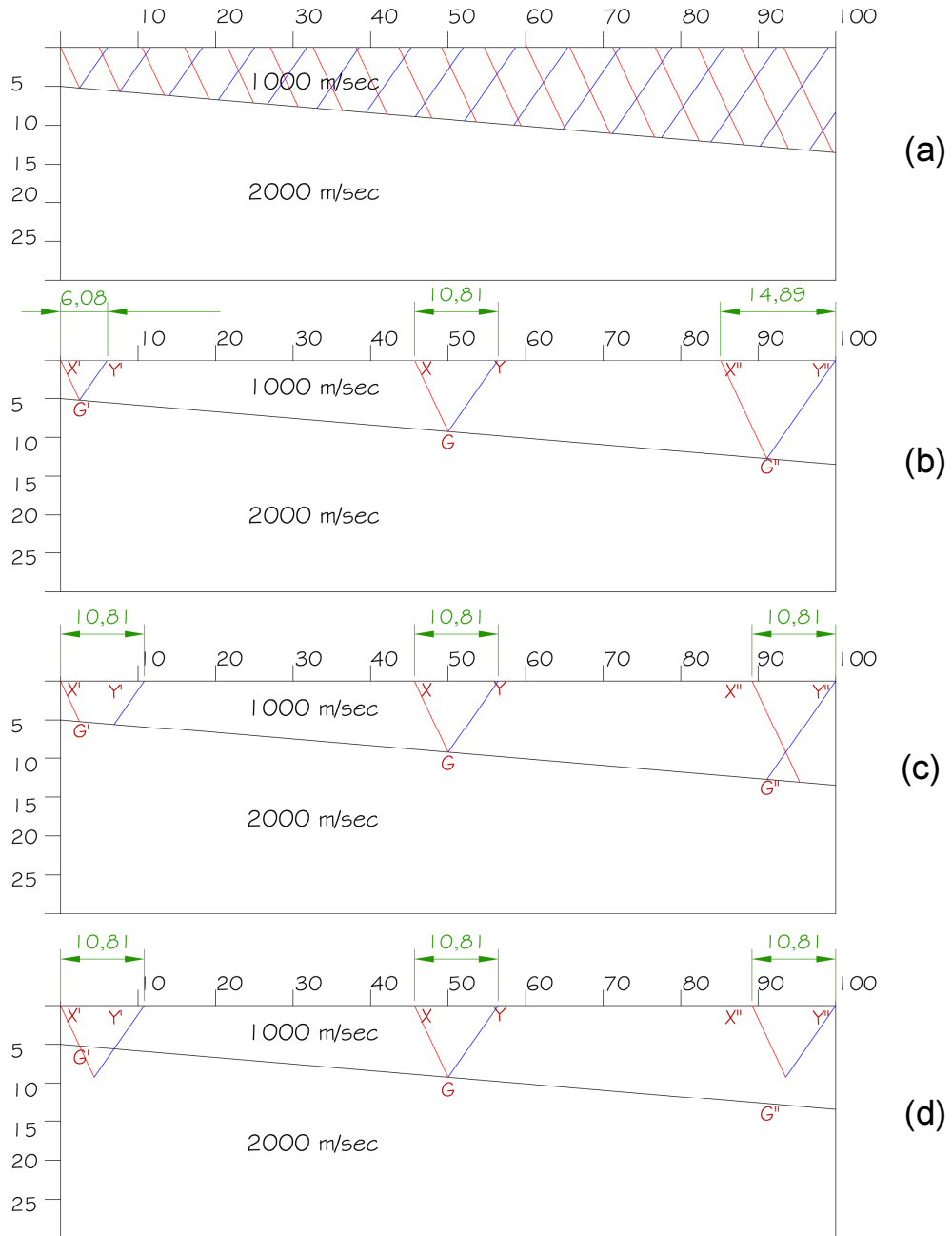


Figure 1: (a) Different optimum XY spacing from the same refractor, (b) Three different values of XY, at three different locations on the same refractor, (c) Three variable values on the refractor from which rays emerge when the optimum XY is selected as 10.8 m and, (d) Three different depths from which the rays will emerge when the optimum XY is selected as a 10.8 m.

The rays emerging from refracting interfaces depend mainly on the depth from which they are refracted, the velocity contrast and the dip angle of these refracting interfaces. Figure 2 shows that the  $X'Y'$ ,  $XY$ , and  $X''Y''$  separations increase with increasing dip angle of the refractor; from about  $5^\circ$  -  $8^\circ$   $30'$ . So how can one hope to find an optimum  $XY$  spacing with a constant value in such case?

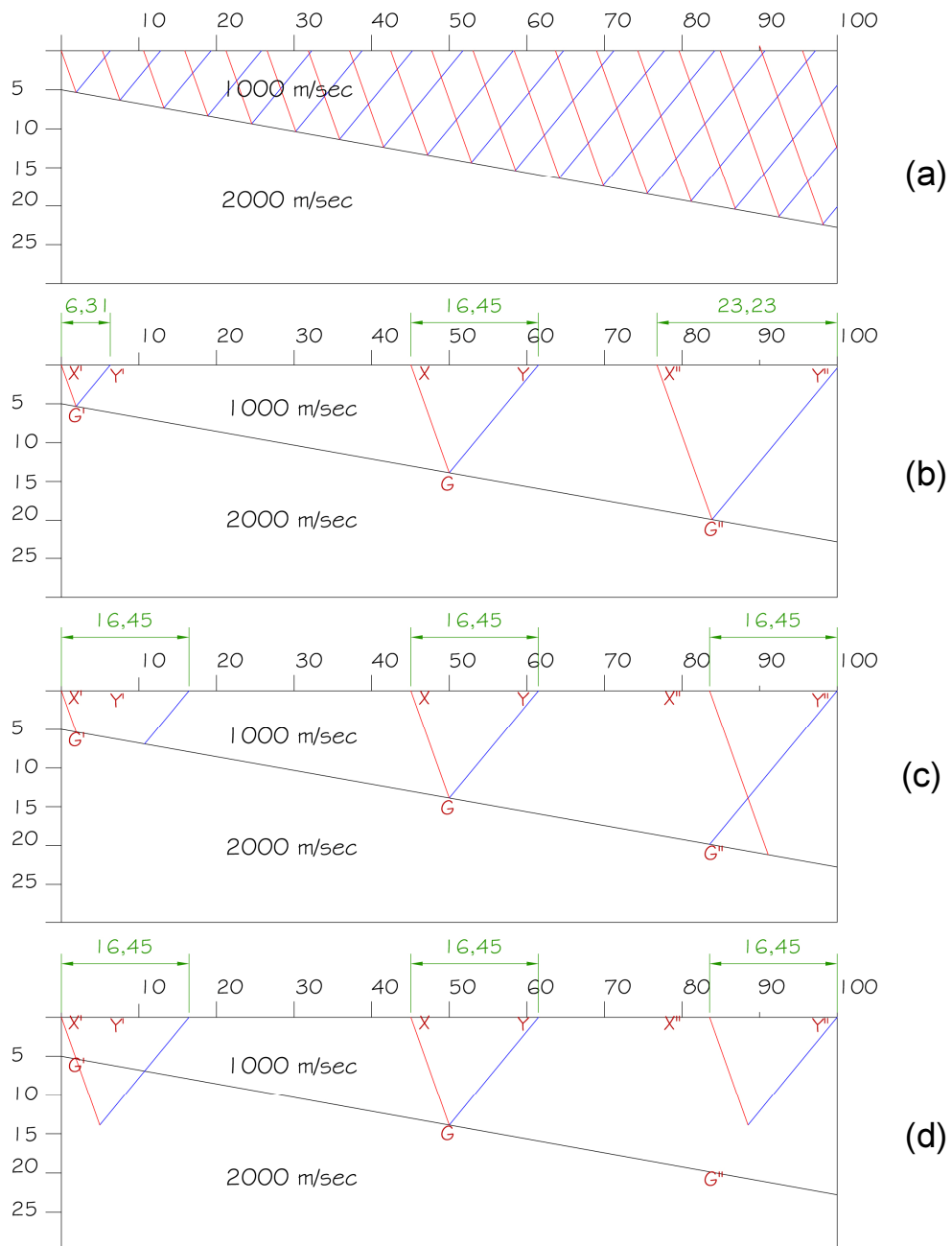


Figure 2: The  $XY$  values increase with the increasing of the dip angle of the refractor.

## Second case:

### A step or a fault model (Palmer, 1986, p.64)

In case of a step or a fault model, the first arrival traveltime curve has three different traveltime elements recorded from three different refracting interfaces from forward and reverse directions (Fig. 3). Applying Snell's law, it will be found that:

- 1- The optimum X'Y' spacing along the refracting interface 2 is constant and equal to 4.36 m.
- 2- The optimum X''Y'' value spacing for the refracting interface 4 is also constant and equal to 13.09 m. and
- 3- The refracting interface 3 shows variable values of XY.

There is a unique XY value for each two rays emerging from the same point on the horizontal refracting interfaces (2 & 4) due to the vertical depths  $d_1$  and  $d_2$  (of the refracting interfaces 2 and 4, respectively) according to the following relations:

$$X'Y' \text{ separation} = 2d_1 \times \tan i_c = 4.36 \text{ m}$$

$$X''Y'' \text{ separation} = 2d_2 \times \tan i_c = 13.09 \text{ m}$$

where  $d_1$  and  $d_2$  are equal to 5 and 15 m, respectively.

Accordingly, there is no single constant value that can be accepted as optimum spacing for the three refracting interfaces. The use of 10 m. as optimum XY value (as given by Palmer, 1986, p. 100) will be found on a single point on the refracting interface 3. At all other points on the refracting interfaces, all rays with XY = 10 m will emerge from two different points either below or above the refracting interfaces or from two different points (but not near the same point) on the refracting interface. In other words, it is impossible to find emerging rays from these refracting interfaces with a single XY value equal to 10 m (optimum XY separation as suggested by Palmer 1986).

In his book (1980), a step model is used and the relation between the velocity analysis function and the optimum XY spacing was graphically given as shown in Fig. 4. However, in his another book (Palmer, 1986, p.66) the above mentioned model was presented and the optimum XY spacing is given as 10 m. (Fig. 5). It is so strange that for the same model, two different results are obtained. This means that the velocity analysis function can be changed over the years. The main reason for this is due to the absence of a constant XY spacing. As shown above, the rays emerged from the same points on the refracting interfaces 2 & 4 have two different optimum XY values while the refracting interface 3 has variable XY separations (Fig. 3b).

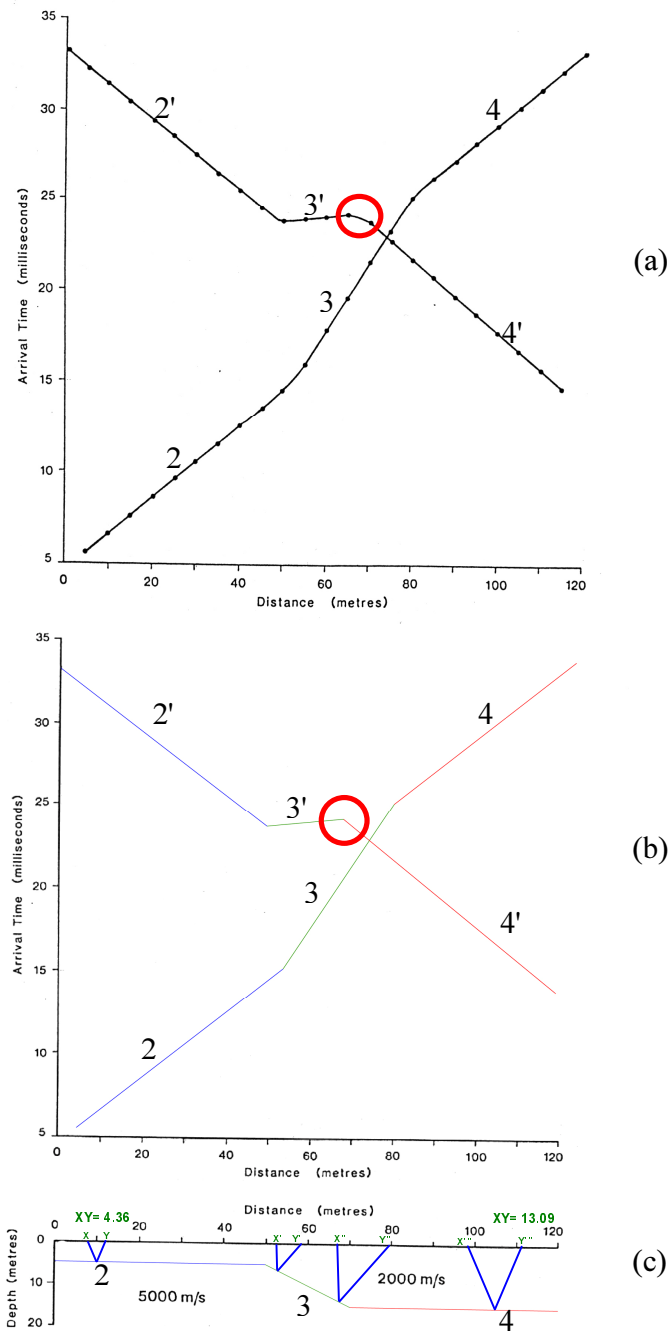
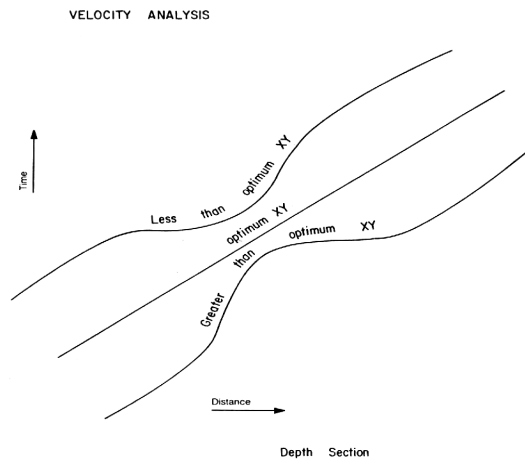
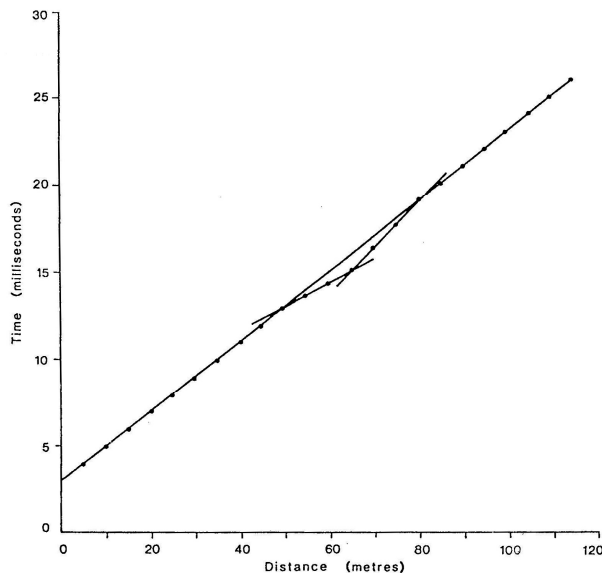


Figure 3: A step or a fault model. The two horizontal refracting interfaces (2&4) have two constant XY spacing while the refracting interface 3 shows variable XY values (Palmer, 1986, P.64). Note the connection of the two refracting interfaces 4'&3' in (a) and (b).



(a)

Figure 4: Schematic representation of velocity analysis function for a refractor with a step in depth. (Palmer, 1980, P. 34)



(b)

Figure 5: The velocity analysis function does not determine the optimum XY spacing for the same model. (Palmer, 1986, p. 66)

### Third case:

#### Irregular refractor surface (Palmer, 1980, p. 17)

In this case, the refractor consists of several refracting interfaces. The emerged rays from each refracting interface have different optimum XY spacing (Fig.6). Consequently, the optimum XY values are different. They range from about 12.1 - 20.2 m. In case of the refracting interfaces have the same velocity and variable layer thickness (or depth), the optimum XY values will be also variable (12.1 m at refractor depth of 15 m at  $S_1$  and 20.2 m. at depth of 25 m at  $S_2$ , respectively). If the refracting interfaces have the same thickness but vary in velocities, then the rays emerged from points at the same depth will have different optimum XY values. The points  $S_2$  and  $S_3$  have the same depth (25 m), but the emerged rays from these points have two

different optimum XY values (equal to 16.2 and 15.7m., respectively) as a result of the variations in velocity along these refracting interfaces and consequently the critical angle (Fig. 7b).

Palmer deduced from the velocity analysis data curves (1980, Fig.6 p.17) that the XY spacing of 20 m can be used as an optimum spacing. It is clear that the rays of optimum separation of 20 m will emerge from different points on the refracting interfaces (but not near the same point). At this optimum XY spacing (20 m) the refracted rays will merge from either above or below the refracting interfaces.

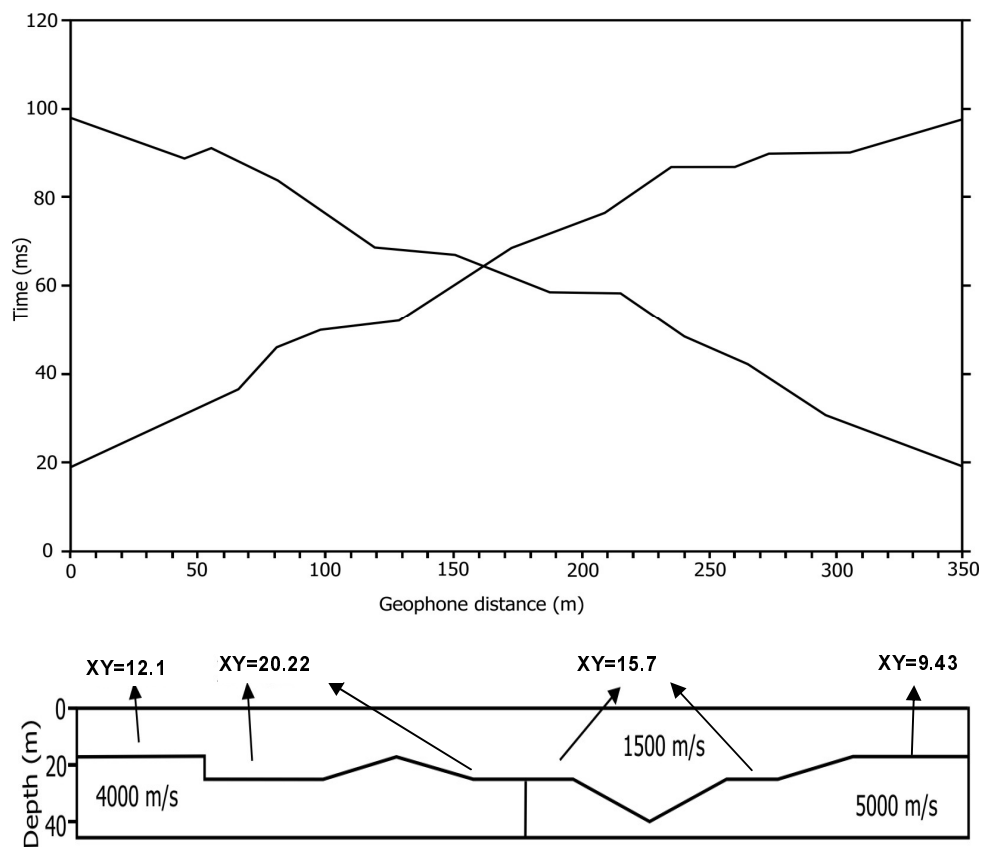


Figure 6: Different XY values are recorded from different refracting interfaces as a result of change in depth or change in velocity contrast (for rays refracted from the same depth).



#### Fourth case:

##### Irregular ground surface (Palmer 1980, p.20)

In case of an undulating topography, the rays at the ground surface and emerged from the same points on the refractor (even a horizontal one) have two different optimum XY values (Fig. 8). The studied model shows that the XY spacing has different values (Fig. 8b). The given optimum XY spacing by Palmer (1980 , p. 21 was zero). At zero-XY distance, the rays emerge from two points on the refractor and the rays have a surface common point (zero spacing) on the ground surface. In (Fig. 8b) two different rays (red in color) from two different refracting interface with two different velocities to get zero optimum XY value.

If the optimum XY distance; attained when the rays at the ground surface have emerged from near the same point on the refractor, this will indicate that there is no clear differentiation between the GRM and the conventional reciprocal time methods (Hawkins, 1961). No wonder, Whiteley (2006) considered that GRM method is a restriction of the reciprocal method.

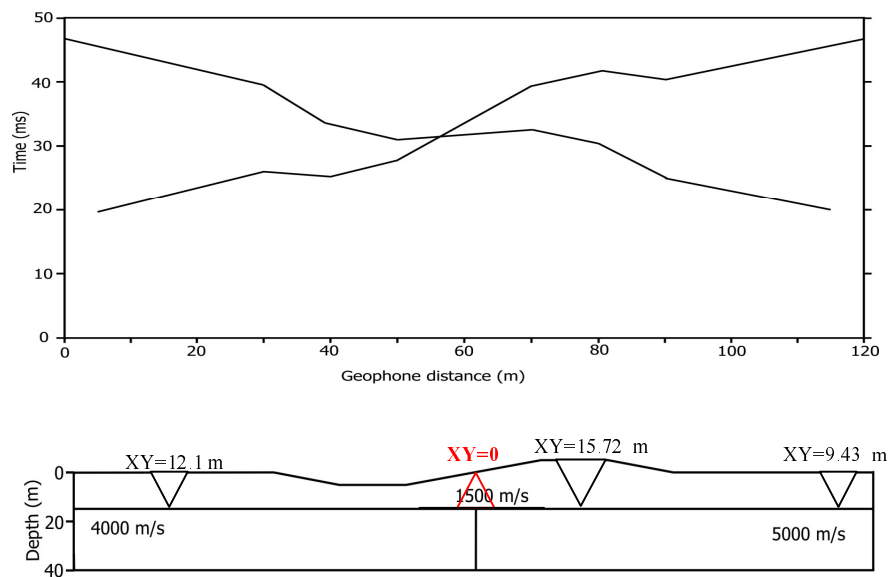


Figure 7: Different XY values recorded from the two refracting interfaces as a result of elevation change and the change of the velocity contrast (If they have the same vertical elevation). The use of zero offset (red color rays) shows that these two rays emerge from two different refracting interfaces.

## Discussion and conclusions

In this study, four cases and five models representing simple subsurface structures are used to show the credibility of the optimum XY separation. The first case includes two models of two plane dipping layers. The last three models are extracted from Palmer (1980 & 1986). The optimum XY spacing with rays emerging from the same point on the refractor is restricted to be found only in a two horizontal layer with constant velocities. In all other cases, the optimum XY spacing has neither constant value nor emerging from the same point on the refractor. On the contrary, most of XY values have different spacing. Taking into account of these considerations, it is clear that the GRM is based on empirical observations i.e. is theoretically unsubstantiated. Therefore, the use of some non-scientific terms in this method such as an *optimum*, *near the same point* on the refractor, showing *maximum smoothness* or *less structure* do not mean anything since they are empirically defined. At the same time, the derivation of the so called optimum XY spacing in the following way "*the optimum XY can be derived from the velocity analysis function curves computed for a range of XY separations and then with manual curve fitting over discrete intervals*" shows lacking of any mathematical derivation.

The refracted rays from the same point leave a dipping interface will have variable XY spacing (as measured along the earth's surface) with both depth (i.e. at the point from which the are emerging on the refractor) and dip angle. The XY spacing increases continuously with depth along a dipping refractor. Therefore, there is no single XY value for a dipping interface can be found. Moreover, the velocity analysis plot cannot be used to extract an optimum XY from the reversed traveltimes data using lateral migration (as suggested by Palmer 1980). It is true that there is a single XY value at every depth for a dipping refractor, but the point of emergence is not centrally located between the points of arrival at the surface i.e. at X and Y. Increasing the dip angle migrates this point in the up-dip direction.

The shallow seismic refraction technique has inherent problems, such as the undetected layers, ambiguities, first and later arrivals....etc. Other main problems are related to the interpretation techniques themselves. Most of them are restricted to simple models, but the generalization invoking lateral data migration in the GRM is responsible for false interpretations. Since the GRM incorporates heuristic assumptions and has no mathematical basis, additional uncertainties are raised in its application, even in case of a simple models as discussed before. The lack of constant XY spacing in all cases (except the horizontal ones) will lead to more uncertainties in its application.

In the GRM determination of the "average velocity" in the overburden above the "main refractor" (assuming it exists), the relationship between the absolute measured traveltimes and the actual

overburden velocities is ignored. This means that computed traveltimes in the overburden using the GRM average velocity rarely match the actual times. When this happens Palmer suggests that hidden layers and/or velocity inversions are present but does not prove this. In case of these undetected layer problems, there are no recorded data as first arrivals on the traveltime curve and the GRM relies completely on its “averaged velocity” obtained at a single XY value. So the question must be asked, how can the GRM produce accurate interpretations without first arrivals recorded from the shallower refracting interfaces? If the application of the GRM on simple models is a doubtful matter, then the use of it in relative complex structure or where seismic anisotropy is present will be also in doubt.

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